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ON THE

CONSTRUCTION

OF

IRON ROOFS.

A Theoretical and Practical Treatise,

BY

FRANCIS CAMPIN, C.E.,

Mem. Nat. Acad. Gt. Britain; F.A.S.; Past President Civ. and Mec. Eng. Soc.;
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ETC. ETC. ETC.

WITH WOODCUTS, AND PLATES OF ROOFS LATELY EXECUTED.

NEW YORK:
D. VAN NOSTRAND, 192 BROADWAY.
1868.

Eng 17.68
6631

JUN 20 1917
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J. HAWKSHAW, Esq., *C.E.*

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PREFACE.

HAVING already issued two volumes of Examples of Iron Roofs, it has occurred to the Author of this work, that in order to render it complete some account should be given of the Principle upon which the construction of Iron Roofs depends; and this is the more necessary, as the subject does not appear hitherto to have been treated in accordance with its merits. We have, therefore, in the following pages explained as succinctly as is compatible with the nature of our subject the theory of lattice girders, trusses, and arches, as applied to roof structures, separating the purely theoretical from the practical theoretical part; furnishing in the former a complete investigation of the nature of such structures, and in the latter giving the ultimate formulæ in their simplest forms, in order to render them easy of reference, and to form also a Supplementary Volume to the two volumes of Executed Examples of Iron Roofs.

In conclusion, it may be desirable to direct the reader's attention to the fact, that the formulæ for strain are also applicable to Timber Roofs, as indicated in the last paragraph of the present volume.

ON

THE CONSTRUCTION OF IRON ROOFS.

PART I.

INTRODUCTION.

THE rapidity of the progress made during the last few years in the construction of iron roofs, has placed this branch of the iron manufacture far in advance of the existing treatises on iron construction. We therefore propose in the following pages to place before our readers a complete, but concise, account, theoretical and practical, of iron roofs.

Roofs may be divided into two principal classes: roofs proper and domes. In the former class we include those supported by arched structures, lattice girders, and trusses. The second comprises the various forms of domes. The greater number of roofs belong to the first class, and we find that roofs of small span are usually carried by trusses simply, whereas, in the case of larger roofs, the small trusses are sometimes supported by lattice girders. Arches are also used for roofs of considerable span.

With regard to the materials and arrangements employed in the coverings of roofs, it is unnecessary in the present volume to consider them, as these matters of detail have been fully shown in the previous volumes of the present work; hence the ribs, principals, or girders, only require to be treated of.

The general points to be considered in the construction of roofs, resolve themselves into two principal ones: the weight of the roof and the means of ventilation. With the first point we have especially to deal, and it is necessary also to consider the second, as far as it affects the main supports of the roof; that is to say, forms must be examined which admit of ventilation by various means.

The weight of the roof will, of course, depend upon the load, and upon the manner of applying the same. The load is constant, necessarily, and for practical purposes it may be regarded as uniformly distributed upon its supports.

The load is also at rest, hence the girders or trusses, not being subjected to concussions or vibrations, may be calculated for greater resistance than ordinary bridge girders.

In considering the mode of calculating the strains upon the different parts of the roof structures, great care is necessary in order to distinguish perfectly between the lattice girder and the trussed principal, for at first sight it might appear to many that the latter is similar in principle to the former; but it will be shown that such is not the case, as in the lattice girder the struts incline downwards from the centre of the girder towards the points of support; whereas in the trussed principal they are usually inclined in the opposite direction.

It will be necessary, therefore, to regard the lattice girder as a whole, and the trussed principal as consisting of a number of trusses or semi-trusses, of which the smaller are carried by the larger; so that in calculating the strains upon this arrangement, they will be traced through the smaller trusses to the larger ones.

With the arched structures no difficulty will be found, the principle being identical with that of a common linear arch, the points to be considered in the construction of which are two: the direction of the strain, and the intensity of the same; it being necessary so to design the form of the arch that no bending moment shall at any point be brought to bear upon it, so that it may be subject to no strain except direct thrust.

We will now conclude these introductory remarks, and pass on to the theoretical considerations connected with the construction of iron roofs and domes.

PART II.

THEORETICAL.

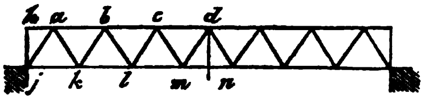
In the present part it is proposed to examine analytically the strains upon the various elements used in roof construction, and in so doing, the matter will be considered in a purely theoretical sense, the formula being reduced to a practical condition in a subsequent part.

The first form which will be considered is the lattice or triangular girder. The following notations will be used throughout the present work:—

l = Span of roof in feet.
w = Weight per lineal foot on girder, truss, or arch.
n = Number of triangulations in a girder.
s = Distance in feet between the apices of two consecutive triangles of one triangulation.
λ = Length of a lattice bar, strut, or tie, in feet.
d = Depth of girder in feet.
v = Versine, or greatest rise of arch in feet.

Other notations required will be specified hereafter.

Fig. 1.



Let Fig. 1 represent a triangular girder, consisting of two flanges or horizontal members connected by a single triangulation, comprising seven triangles, as shown. Let these be isosceles triangles; that is to say, if we take any triangle $b\ k\ l$, let two sides be equal, and let these sides be those which are inclined to the flange, so that $b\ k$ is equal to $b\ l$; it will then follow that the angle $b\ k\ l$ is equal to the angle $b\ l\ k$. Call the value of this angle a.

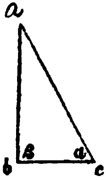
s will be equal to $a\ b$, $b\ c$, $c\ d$, etc., n will be equal to one, or unity, also l is equal to $7\ s$.

Let us now determine the strain upon each bar of this girder. The girder is in form symmetrical, it is also symmetrically loaded, hence the strains on each half of the girder will be similar, and each half of the girder may be regarded as carrying half the total distributed load; hence it will only be necessary to examine the strains on one half of the girder. Before proceeding further, it may be desirable to mention the offices of the different parts of the girder. The load is transmitted from the open space on to the points of support by the lattice bars, which constitute the web, and this web is retained in position or stretched by the top and bottom horizontal members or flanges. The load will be brought upon the girder at every point of junction, *a b c*, etc., and the load on the apex of each triangle will be

$$= w\,s$$

We will now proceed to resolve the strains upon the bars, which may readily be done by means of the well-known principle of the parallelogram of forces; but we shall here for convenience employ trigonometrical terms in place of linear measurements, in stating the strain. The trigonometrical terms used are those commonly employed for the solution of plane triangles. Let *a b c*, Fig. 2, represent a plane triangle, the side *a b* being known, and the angles α and β, it is required to determine the side *a c*, which may be found from the formula

Fig. 2.



$$a\,c = \frac{a\,b\,.\sin\beta}{\sin\alpha},$$

which it is unnecessary here to demonstrate, as the explanation of it may be found in any elementary work on trigonometry. The formula may of course be applied to any two sides of a triangle and their subtending angles. If the angle β be a right angle, the above formula will become

$$a\,c = \frac{a\,b}{\sin\alpha}$$

because

$$\sin\beta = \sin 90^\circ = 1$$

Commencing at the centre of the girder, and proceeding towards the extremity *k*, we have first to determine the strain upon the bar *d m*. This bar is loaded with half the weight upon the apex *d*, so that the load upon the bar

$$= \frac{w\,s}{2}$$

The strain will be found by resolving this weight in the direction of $d\,m$. It will be compressive and

$$= \frac{w\,s}{2 \sin a}$$

At the point m, this strain will be resolved into two others: one acting upon $m\,c$, the other acting upon $m\,n$. The parallelogram for solving these forces is found in the girder, being formed by two triangles; so that if $d\,m$ equals the strain on $d\,m$, $c\,m$ will be that on the bar $c\,m$, and $c\,d$ that on $m\,n$. The strain on $c\,m$ will be tensile, and

$$= \frac{w\,s}{2 \sin a}$$

Thus we see that the strain on any tie is equal to that upon the strut which precedes it. This strain will necessarily be transmitted through every bar to the abutment j; or, in other words, the load on the bar $d\,m$ will act on every other bar to the abutment, for the office of the bar $d\,m$ is to transmit the load from the point d to the point m, whence the tie $c\,m$ transmits it to the point c, so that every strut must support all the weight between itself and the centre of the girder. The load on $c\,l$ will therefore be

$$= \frac{w\,s}{2} + w\,s = \frac{3\,w\,s}{2}$$

and the strain on the same, and on the tie $l\,b$

$$= \frac{3\,w\,s}{2 \sin a}$$

Summing the strains for all the diagonals, the following series is obtained, for any girder, commencing at the centre, and proceeding towards either point of support:—

On first strut and tie	$\frac{w\,s}{2 \sin a}$
On second " "	$\frac{3\,w\,s}{2 \sin a}$
On third " "	$\frac{5\,w\,s}{2 \sin a}$
On mth " "	$\frac{(2\,m-1)\,w\,s}{2 \sin a}$

This last holds good for any bar distant m struts from the centre, except the last, $a\,j$, where there is a slight distance, as the load between a and h is borne partly by the strut and partly by the standard $h\,j$, so that the load on the summit a is in fact

$$= \frac{3\,w\,s}{4}$$

This difference will, however be neglected, and $w\,s$ will be taken as the load on this summit, in common with those on the others.

Having disposed of the diagonals, it remains to consider the strains upon the horizontal members. Let us commence with the bottom, or tension member.

If Σ represents the strain on any strut, except the extreme ones, this strain will, as before stated, be transmitted to the following tie, and to the bottom horizontal member; and referring to Fig. 1, we see that the strain upon the bottom member due to that upon the strut may be expressed trigonometrically as follows, the strain being

$$= \Sigma \,.\, \frac{\sin 2\,a}{\sin a}$$

because the sine of an angle is equal to the sine of its supplement, or

$$\sin a = \sin\,(180^\circ - a)$$

By an ordinary transformation the previous expression becomes

$$= \Sigma\, 2 \cos a$$

but calling W the total load upon the strut

$$\Sigma = \frac{W}{\sin a}$$

and the horizontal strain

$$= W\,\frac{2 \cos a}{\sin a}$$

$$= W\; 2 \;\mathrm{cotan}\; a$$

This is the strain which will be brought upon the bottom or tension member at every joint except the extreme ones. Let us now examine the strain produced by the extreme struts. Let Σ' represent the direct strain upon the extreme strut $a\,j$, then the strain produced upon the tension member will be as follows, the strain being in this case resolved vertically upon the pier, and horizontally upon the tension member:

$$\Sigma' .\ \cos.\ a$$

this is the direct strain upon *j k*, but

$$\Sigma' = \frac{W}{\sin a}$$

hence the foregoing expression becomes

$$= W \cdot \frac{\cos a}{\sin a}$$
$$= W \cdot \text{cotan}\ a$$

We have now found expressions for the strain upon any bar of the bottom horizontal member, and it is a matter of observation that the strain increases from the piers toward the centre of the girder; for commencing at *j* there are increments of strain at *k*, *l*, *m*, &c., arriving at a maximum at the centre where the strains produced by the load on one side of the centre are, as it were, met and equilibrated by those produced by the load upon the other side of the centre of the girder. The strain at any given joint may be readily found by summation, thus:—Let the girder contain nine triangles; there will then be four and a half on each side of the centre, or five struts and four ties in one half of the girder, the loads upon the ties and struts will be

$$\text{On the first, a load} = \frac{w\,s}{2}$$
$$\text{On the second ,,} = \frac{3\,w\,s}{2}$$
$$\text{On the third ,,} = \frac{5\,w\,s}{2}$$
$$\text{On the fourth ,,} = \frac{7\,w\,s}{2}$$
$$\text{On the fifth ,,} = \frac{9\,w\,s}{2}$$

commencing at the centre, and passing towards one of the points of support. The strain at each joint of the tension member will be, commencing at one point of support, and passing towards the centre of the girder,

$$\text{First joint, a strain} = \frac{w\,s\ 9\ \text{cotan}\ a}{2}$$

$$\text{Second ,, ,,} = \frac{w\,s\ 9\ \text{cotan}\ a}{2} + w\,s\ 7\ \text{cotan}\ a$$
$$= \frac{w\,s\ 23\ \text{cotan}\ a}{2}$$

$$\text{Third joint a strain} = \frac{w\,s\,23 \text{ cotan } \alpha}{2} + w\,s\;5 \text{ cotan } \alpha$$

$$= \frac{w\,s\,33 \text{ cotan } \alpha}{2}$$

$$\text{Fourth ,, ,,} = \frac{w\,s\,33 \text{ cotan } \alpha}{2} + w\,s.\,3.\text{ cotan } \alpha$$

$$= \frac{w\,s\,39 \text{ cotan } \alpha}{2}$$

$$\text{Fifth ,, ,,} = \frac{w\,s\,39 \text{ cotan } \alpha}{2} + w\,s.\text{ cotan } \alpha$$

$$= \frac{w\,s\,41 \text{ cotan } \alpha}{2}$$

A course precisely similar to the above may be used for the top or compression member, but it must be borne in mind that in this case the strain is transmitted by the ties to the horizontal member; and as there is no extreme tie corresponding to the end strut, the one expression

$$W\,2 \text{ cotan } \alpha$$

will apply to every joint in the top flange. The strain at every joint, commencing, as before, at the point of support, will be as follows :—

$$\text{First joint a strain} = w\,s\;\;7 \text{ cotan } \alpha$$

$$\text{Second ,, ,,} = w\,s\;\;7 \text{ cotan } \alpha + w\,s.\,5.\text{ cotan } \alpha$$

$$= w\,s\;12 \text{ cotan } \alpha$$

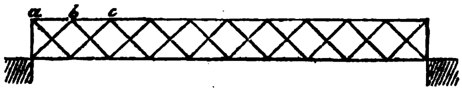
$$\text{Third ,, ,,} = w\,s\;12 \text{ cotan } \alpha + w\,s.\,3.\text{ cotan } \alpha$$

$$= w\,s\;15 \text{ cotan } \alpha$$

$$\text{Fourth ,, ,,} = w\,s\;15 \text{ cotan } \alpha + w\,s.\text{ cotan } \alpha$$

$$= w\,s\;16 \text{ cotan } \alpha$$

Fig. 3.



By means of the set of formulæ now completed, we can readily compute the strain on every part of any given triangular girder of the series of triangles; the method of applying these formulæ in practice will be described subsequently.

If we have more series of triangle than one, the weight must be found for the summit of the triangle of the series, and the strains computed on one series of diagonals as before; and the same with the other series. The strains on the flanges will, as before, be found by summing the strains produced at each joint.

Fig. 3 shows a girder with two series of triangles. It will be seen that one terminates with a strut and the other with a tie; and it will be advisable in such a case to calculate separately the strains on each series. But when there are many triangulations, the strains on one may be regarded as equivalent to those on the others, and the labour of calculating the strains for every series will thus be avoided.

To find the load on any apex, multiply the weight per foot run, by the distance between two successive joints. Thus, let *z* equal the distance between two joints, *a* and *b*, Fig. 3; then the load on each apex will be

$$= w z$$

In resolving the strains produced on the flanges by one series, it must be borne in mind that in one series the terminal bar is a strut, and in the other it is a tie; hence, in this case, the strains are exactly the same on both flanges.

We have now concluded the account of the lattice girder, so far as it is used in the construction of roofs, and will now pass on to examine the principles of simple and compound trusses.

Fig. 4.

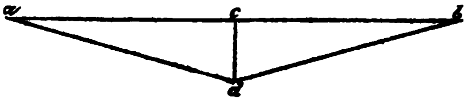


Fig. 4 represents a simple truss; it consists of a horizontal beam, *a b*, a strut, *c d*, at the centre, and two tension rods, *a d* and *b d*, the whole being supported at *a* and *b*. It will be necessary in the first instance to consider the strains produced by a load concentrated at *c*. Half this load will be borne, half by the point of support at *a* and half by that at *b*, and these halves must respectively pass through the tension rods *d a* and *d b*. Let *i* = the angle *c a d*, and also = the angle *c b d*.

Let W = the weight acting at the point *c*; then the thrust upon the strut *c d* will be equal to W, and the load on each tie bar

$$= \frac{W}{2}$$

The direct strain upon either tie bar may be found by a method analogous to that used for the triangular girder already described. It will be

$$= \frac{W}{2 \sin i}$$

The strain in compression upon the horizontal member, produced by the action of the tension rods, will be

$$=\Sigma \cos i.$$

where Σ equals the strain upon the tension rod; but

$$\Sigma = \frac{W}{2 \sin i}$$

hence the horizontal strain is

$$= \frac{W}{2} \cdot \frac{\cos i}{\sin i}$$

$$= \frac{W}{2} \cdot \operatorname{cotan} i$$

In the case of an inverted truss, the strains are exactly the same in intensity, but they differ in kind: that is to say, where there is a tensile strain in the inverted truss there is a compressive strain in the upright truss, and *vice versa.*

With regard to a distributed load, the following conditions may be observed:

The load between *a* and *c* is half of it borne by the pier *a* direct, and half of it is transmitted through the strut *c d*; hence in this case the direct strains are half what they would be if the load were concentrated; therefore if W equals the total load, the strains will be as follows:—

Strain on strut *c d* $= \frac{W}{2}$

Strain on tension rod $= \frac{W}{4 \sin i}$

Strain on horizontal member $= \frac{W}{4} \operatorname{cotan} i$

Fig. 5.

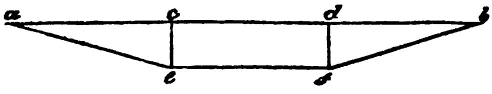


Fig. 5 represents another form of truss, somewhat similar to the former in principle, but with the tension-rods so arranged that the strain upon them is proportionately less.

Let *i* equal the angles *c a e*, or *d b f*, and let the truss be loaded with two weights, each equal to W; placed, one at the point *c*, and the other at the point *d*. Then the strain on either of the struts *c d*, *e f*, will be

$$= W$$

The strain on either of the tension rods *a e* and *f b* will be

$$= \frac{W}{\sin i}$$

The compressive strain upon the horizontal member *a b*, and the tensile strain on the tension rod *e f*, will each be

$$= W \cotan i$$

The case of an equally distributed load must next be taken. The above formulæ will apply if W be substituted by the load per lineal foot multiplied by half the distance *a c*, plus half the distance *c d*, or if $x = a\,c$ and $y = c\,d$

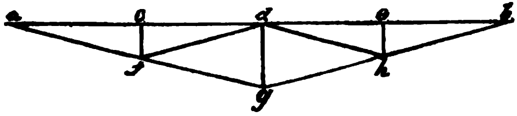
$$W = w \,.\, \frac{x + y}{2}$$

In the case of a distributed load, it is taken for granted that the top beam of the truss possesses sufficient transverse resistance to sustain the load between the points of junction with the other parts of the truss.

If the truss shown in Fig. 5 be inverted, the tensile and compressive strains will be reversed.

Simple trusses having been disposed of, it now remains for us to consider the action of more complicated trusses, in which, although the same principles will be found to exist, other points will require consideration.

Fig. 6.



In Fig. 6, *a b* represents the main compression member of a truss, supposed to be supported at each extremity; the primary truss is formed by the compression member *a b*, the central strut *d g*, and the tension bars *g a*, *g b*. This primary truss supports two smaller or secondary trusses, one of which consists of the part *a d* of the compression beam, with the strut *c f*, and the tension rods *f a*, *f d*, *f a* being a portion of one of the tension rods of the primary truss. On the other side of the primary truss we have another secondary, *d h b*, similar in form to that already described. From inspection it is found that in this arrangement certain bars act as parts of different trusses, hence these bars will have a variety of offices to perform; this, however, will not materially complicate the subject, as it is evident that the ultimate strain on any bar will be the sum of all the tensile strains, minus the sum of all the compressive strains, or the contrary. Let us now proceed to determine the strains upon each element of the truss. The load is supposed to be equally distributed over the horizontal member of the truss, calling *w* the load per lineal foot on the truss, and *s* the distance, *a c*, which is equal to *c d*, *d e*, or *e b*, the struts being placed at equal distances along the horizontal member, it will follow that the load will be distributed as follows:—At the point of support, *a*, there will be half of the load between *a* and *c*, which

$$=\frac{w.s}{2}$$

and on each of the points *c d e* there will be a load

$$= w\ s$$

On the point of support, *b*, there will be a load equal to that on *a*. It now remains for us to determine the effect of these loads; that is to say, it remains for us to calculate the intensity of the strains produced by them. The strains upon the struts will evidently be equal to the loads upon the same, the whole load on each strut being necessarily transmitted through it to the two points of support, thus the strain on the strut *c f* will be

$$= w\ s$$

This strain is transmitted to the points *a* and *b* through the tie bars *f a* and *f d*. That portion which is borne by the point *a* passing through the rod *f a*, and that which is borne by the point *b* passing through the rod *f d*, it is evident that we must, in the first place, determine the proportions of the load borne by each point of support. This may be accomplished by the well known laws of the lever, the result of which will show that the proportions of the weight on each point of support will be inversely as the distances of these points from the weight. Thus calculating the pressure on *b*, we have a quantity

$$= w\ s\ \frac{a\ c}{a\ b}$$

and on the point *a* we have a load

$$= w\ s\ \frac{c\ b}{a\ b}$$

and these quantities will be the *loads* carried by the tie bars, the lesser being supported by *f d*, and the greater by *f i*.

By the construction of this truss the angles *a c f*, *c d f*, *e d h*, *e b h*, are all equal to each other. Let the value of one of them be represented by *i*, then the strain on the bar *a f* will be

$$=\frac{w\ s}{\sin.\ i}\ \frac{c\ b}{a\ b}$$

and that on *f d*

$$= \frac{w\,s}{\sin i}\ \frac{a\,c}{a\,b}$$

From these formulæ it appears that the method of calculating the strain on this secondary truss is similar to that employed in the calculation of a simple truss, but with this difference: in the present case the loads to be carried by the tie bars are unequal, depending, as they do, upon the position of the strut which the tie bars support.

The strains on the other secondary truss, *d h b*, are similar to those on *a f d*, the strain on *h d* being equal to that on *f d*, and that on *h b* equal to that on *f a*.

We now come to consider the strains on the primary or main truss, and in order to arrive at these we must find the strains produced by that part of the load which acts directly upon this truss, and also the strains which are transmitted from the secondary trusses, commencing with the strains produced by the direct action of the load, we find on the strut *d g* a thrust

$$= w\,s$$

This strut has also to support the loads brought upon it by the ties *f d* and *h d*, which will be equal to twice the load on *f d*, but the load on *f d* was found to be

$$= w\,s\,\frac{a\,c}{a\,b}$$

hence the strain brought upon the strut *d g* by the ties *f d*, *h d*, will be

$$= 2\,w\,s\,\frac{a\,c}{a\,b}$$

The total strain upon the strut *d g* will be the sum of the foregoing strains, which

$$= w\,s + 2\,w\,s\,\frac{a\,c}{a\,b}$$

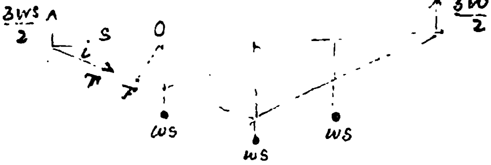
The next step in the calculation will consist in calculating the strains upon the tie bars of the primary truss. We will first determine that upon the bars *g f*, *g h*. The strain on those bars will be due to the thrust upon the main strut, as given above, and the strain on each of the bars *f g*, *g h*, will be

$$= \frac{w\,s}{\sin i}\left\{1 + 2\,\frac{a\,c}{a\,b}\right\}$$

More simply, by moments about O

$\frac{3ws}{2}\times s = T\times OT = T\times s\tan i\cos i = Ts\sin i$

$\therefore T = \frac{3}{2}\cdot\frac{ws}{\sin i}$



The total strain upon the bar *f a* will be found by summing the strains due to the loads on the struts *d g*, and *c f*, as *a f* forms a part of both primary and secondary truss. We thus get for the total strain on *f a*, or *h b*,

$\frac{ws}{\sin i}\left\{\frac{1}{2}+\frac{cb}{ab}+\frac{ac}{ab}\right\}=\frac{3}{2}\frac{ws}{\sin i}$

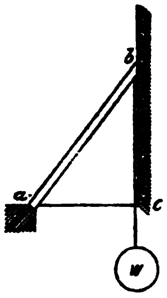
wrong $$\frac{w\,s}{\sin i}\left\{1+\frac{c\,b}{a\,b}+2.\frac{a\,c}{a\,b}\right\}=\frac{ws}{\sin i}\left\{1+\frac{3}{4}+\frac{2}{4}\right\}=\frac{9ws}{4\sin i}$$ This is too large

By reversing any of these trusses the strains are changed from tensile to compressive, and *vice versâ* if the load still acts first upon the same point, viz., the junction of the strut with the horizontal member.

We have now discussed the general conditions of strain in simple and compound trusses fixed in horizontal positions; but it yet remains to examine the effect of load when the trusses are inclined, and this part of our subject is in fact that in which we are most interested, as it will hereafter be seen that in roof structures trusses are very generally inclined.

In order to understand perfectly the mode of action of inclined trusses, it is very necessary to be well acquainted, in the first place, with the ultimate resultants of strain produced upon the supports, and it is therefore deemed desirable to examine the condition of a simple inclined strut.

Fig. 7.



Let *a b*, Fig. 7, represent a simple strut, supported by an abutment at *a*, and *resting* against a wall at *b*, and from the point *b* of the strut let a weight W be suspended. It is evident that the wall at *b* merely prevents the strut *a b* from falling by revolving about the point *a*, and does not afford any vertical support; hence the whole of the load W must be sustained by the abutment *a*. Let *i* = the angle which the strut makes with the horizon, then will the direct thrust upon the strut be

$$= \frac{W}{\sin i}$$

This thrust will (if the strut has no tie at its base) be communicated to the abutment, acting in the same straight line *a b* as in the strut, and there will therefore be two forces acting at *a*, one a vertical pressure tending to *crush* the support, and the other a horizontal force tending to overturn it. Let Σ = the direct thrust upon the strut *a b*, then will the vertical and horizontal forces acting at the point *a* be as follows:—

The vertical force

$$= \Sigma \sin i$$

and the horizontal force

$$= \Sigma \cos i$$

but

$$\Sigma = \frac{W}{\sin i}$$

wherefore the vertical force

$$= \frac{W}{\sin i} \,.\, \sin i = W$$

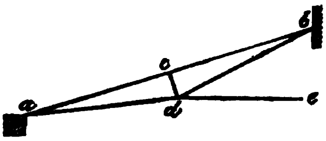
which proves the accuracy of the preliminary remark that the whole of the load must be borne by the support *a*. The horizontal force becomes

$$= \frac{W}{\sin.\ i} \cos i = W \times \operatorname{cotan} i$$

This expression is also equivalent to the horizontal pressure at *b* against the wall *b c*. The horizontal force at *a* may be withstood by a tie *a e*, as shown, which resists the tendency of the foot of the strut to move outward, and in this case the support *a* will only be required to sustain the vertical force, or the weight of the load.

Let Fig. 8 represent a simple inclined truss, acted on at the point *c* by a load W, this load will evidently produce two strains in compression, of which one will act on the strut *c d*, and the other on *c a*; let *e* = the angle made by the strut *c d* with the horizon, *o* = the angle made by *c a* with the horizon, and *i* = the angle *c a d*, = the angle *c b d*. The strain produced by the load W upon the strut *c d* will be

Fig. 8.



all wrong

$$= \frac{W \sin}{\cancel{\sin e}}$$

that upon *c a*

$$= \frac{W \sin o}{\cancel{\sin o}}$$

the tension upon either tie rod will be

$$= \frac{W \sin i}{\sin e} \cdot \frac{1}{\sin \cancel{2} i} \cdot \div 2$$

which will at the points *a* and *b* produce upon the member *a b* strains acting in opposite directions, each

$$= \frac{W}{\sin e} . \cot\text{an}\, i \div 2$$

Hence three strains are produced upon the compression member *a b*, of which two act in a direction towards *a*, and one in the opposite direction; from *b* to *c* there is a strain

$$= \frac{W}{\sin e} . \cot\text{an}\, i \div 2$$

at *c* it is increased to

$$\frac{W}{\sin e} . \cot\text{an}\, i + \frac{W}{\sin o}$$

which passes on to *a*, where it is met, and partially equilibrated by a strain

$$= \frac{W}{\sin e} . \cot\text{an}\, i$$

leaving the point of support *a* to sustain a thrust equal to the difference between the descending and ascending strains, which is

$$= \frac{W}{\sin e} . \cot\text{an}\, i + \frac{W}{\sin o} - \frac{W}{\sin e} \cot\text{an}\, i$$

$$= \frac{W}{\sin o}$$

which is the same as that found upon the lower point of support in the case of a simple strut.

In following this investigation, it must be carefully borne in mind that the truss is only *rested against* the wall at *b*, so that no support can be given at that point in a vertical direction, and it is also supposed that the truss is so formed that the member *a b* does not afford any support by its transverse strength.

The horizontal thrust at *a* may he withstood by a tie *d e*, on which (supposing it to be horizontal), there will be a strain

$$= w \times \cot\text{an}\, o$$

this strain will, of course, be transmitted through the bar, *a d* to *d e*, being produced by the strain

$$\frac{W}{\sin o}$$

which is the resultant of the strains upon $c\,a$, it must be resolved for $a\,d$, when the strain will be found to be

$$= \frac{W}{\sin o} \cdot \frac{\cos o}{\cos u}$$

$$= \frac{W}{\cos u} \,.\, \text{cotan}\, o$$

where u = the angle made by $a\,d$ with the horizon.

The ultimate strain on the tie $a\,d$ will therefore be

$$= \frac{W}{\sin e .\, \sin i} + \frac{W}{\cos u} \,.\, \text{cotan}\, o$$

With trusses of more complex form placed in an inclined position a very similar course is followed out. It may, however, be desirable to illustrate the means employed in this case.

Fig. 9.

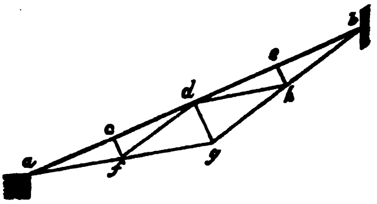


Fig. 9 represents an inclined compound truss loaded uniformly. In order to calculate the resultant strains upon each element, the following method must be employed:—

First find the load on each point c, d, e. It will be equal to the load per foot run, multiplied by the distance $a\,c$, or $c\,d$, or &c. At b and at a there will be loads, each equal to half the foregoing. Let W = the load on c, d, or e, then $\frac{W}{2}$ = the load at a or b, and let i = the angle made by the main compression member $a\,b$ with the horizon; then the strains on that member will be

$$\text{On } b\,e \text{ a strain} \quad = s + \frac{W}{2 \sin i}.$$

$$\text{On } e\,d \quad \text{,,} \quad = s + \frac{W}{2 \sin i} + \frac{W}{\sin i}$$

$$= s + \frac{3\,W}{2 \sin i}.$$

$$\text{On } d\,c \quad \text{,,} \quad = s + \frac{3\,W}{2 \sin i} + \frac{W}{\sin i}$$

$$= s + \frac{5\,W}{2 \sin i}.$$

$$\text{On } c\,a \text{ a strain } = s + \frac{5\text{ W}}{2 \sin i} + \frac{\text{W}}{\sin i}$$

$$= s + \frac{7\text{ W}}{2 \sin i}.$$

$$\text{On the shoe } ,, \quad = s + \frac{7\text{ W}}{2 \sin i} + \frac{\text{W}}{2 \sin i}.$$

$$= s + \frac{4\text{ W}}{2 \sin i}.$$

In all of which equations *s* equals the thrust produced upon the members by the tie bars *g a*, *g b*; those upon *f d* and *h d* act upon the central strut *d g*, and do not affect the main compression member.

Let *e* = the angle made by the struts *f c*, *d g*, and *e h*, with the horizon, then will the strain upon *f c* or *e h*, be

$$= \frac{\text{W}}{\sin e}$$

and the strain upon *d g* will be

$$= \frac{3\text{ W}}{\sin e} \qquad ?\,2$$

which is due to the load on *d*, and to those on *c* and *e*, which transmit strains through the bars *f d*, *h d*, to the strut *d g*.

It now remains to determine the resultant strains upon the tie rods. Let *k* = the angle *e b h*, or *e d h*. The strains upon *h d* and *h b*, due to the load at *e*, will each be

$$= \frac{\text{W}}{\sin e \,.\, \sin k}$$

hence the total strains upon the rods *h d*, *f d*, will be for each

$$= \frac{\text{W}}{\sin e \,.\, \sin k}$$

On *g h* there will be a strain due to that acting on the strut *d g*; it will be

$$= \frac{3\text{ W}}{\sin e \,.\, \sin k}$$

and *h b* will have to bear this strain in addition to the strain brought upon it by the load at *e*, hence the resultant strains will be

$$\text{On } g\,f \text{ or } g\,k = \frac{3\,W}{\sin e\,.\,\sin k}$$

$$\text{On } h\,b \text{ or } f\,a = \frac{W}{\sin e\,.\,\sin k} + \frac{3\,W}{\sin e\,.\,\sin k}$$

$$= \frac{4\,W}{\sin e\,.\,\sin k}$$

From this expression we may also determine the thrust produced by the tie rods upon the main compression member, which thrust has been designated by s. We shall have

$$s = \frac{4\,W}{\sin e\,.\,\sin k}\,\cos k$$

$$= \frac{4\,W}{\sin e} \times \text{cotan } k$$

By replacing in the former equations s, by its value thus obtained the equations of strains upon the various parts of the main compression member will be obtained.

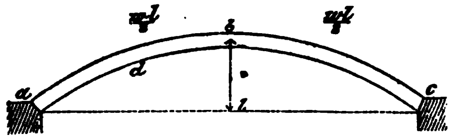
The horizontal thrust at a may of course be withstood by a tie rod attached to g, and in this case the pull upon such tie rod, and the additional strain upon the tie $g\,a$, may be calculated by a method precisely similar to that employed in the preceding case.

The method of extending these calculations in order to apply them to more complicated trusses are sufficiently obvious, as also are the modifications required to render them suitable to trusses varying in form and position from the example illustrated.

The next kind of construction to which we shall refer is the arch form. In this, as in the previous cases, the load is uniformly distributed, and for reasons well known, linear arches intended to support an uniformly distributed load should be of a parabolic form, otherwise there would not be only a thrust on the arch, but bending strains would also occur. The arch, accurately formed, represents the vertex of a parabolic curve, and in general comprises only a very small arc, which will be found to differ but little from a circular arc; so that in arched ribs having any depth, it will be sufficient to make the arch of a circular form, which will contain the true parabolic curve between the extradosal and intradosal surfaces.

Let $a\ b\ c$, Fig. 10, represent an arch, of which l = the span, and v = the rise or versine, w = the load per lineal foot. The first step towards the calculation

Fig. 10.



of the arch will consist in determining the thrust at the crown or centre, which may be found as follows:—We may suppose the load to be gathered into two concrete parts, each equal to half the total, and each being placed half way between the crown of the arch and one of the abutments. Then for the semi-arch $a\ b$, it is found that the load $\frac{w\ l}{2}$ acts about a as a fulcrum at a perpendicular distance $a\ d$, or one quarter of the span from it, hence the moment of the load about the point a will be

$$= \frac{w\ l}{2} \cdot \frac{l}{4} = \frac{w\ l^2}{8}$$

The moment of resistance, or strain, must of course be equal to and counteract the moment of load; but the resistance of the section at the crown acts about a at a perpendicular distance v from it. Hence, calling r = the strain at the crown of the arch, the moment will be

$$= v\ r$$

and because the moments of strain and resistance are equal

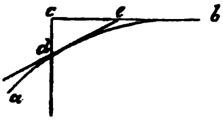
$$\frac{w\ l^2}{8} = v\ r$$

$$r = \frac{w\ l^2}{8\ v}$$

from which the strain at the crown of the arch may be calculated.

An expression for the thrust on the arch at any other point must now be found.

Fig. 11.



Let $a\ b$, Fig. 11, represent a semi-arch. It is required to find the thrust at any point d distant, x feet from the crown of the arch. Let $c\ e$ = the horizontal thrust at the crown, and $c\ d$ = the vertical force of the load between c and b; then the resultant $d\ e$ will be equal to the thrust at d; but $e\ c\ d$ is a right-angled triangle ($c\ e$ being the *horizontal* and $c\ d$ the *vertical* force): therefore

$$\overline{d\ e}^2 = \overline{c\ e}^2 + \overline{c\ d}^2$$

but $c\ e$ = the thrust at the crown, and $c\ d$ = the load between d and the crown, which is

$$= w\ x$$

hence, reducing and replacing, we have for the thrust at any point an expression

$$= \sqrt{\left(\frac{w\,l^2}{8\,v}\right)^2 + (w\,x)^2}$$

$$= w\sqrt{\frac{\cancel{w^2}\,l^4}{(8\,v)^2} + x^2}$$

which may be simplified for the thrust on the bed plates. Let half the total load, or

$$\frac{w\,l}{2} = \mathrm{W} = w\,x$$

in the present case, as

$$x = \frac{l}{2}$$

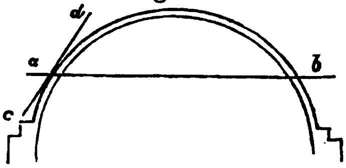
hence, replacing, we get for the thrust on either bed plate.

$$\sqrt{\left\{\frac{\mathrm{W}\,l}{4\,v}\right\}^2 + \mathrm{W}^2} = \mathrm{W}\sqrt{\frac{l^2}{16\,v^2} + 1}$$

These formulæ will be found amply sufficient for the calculation of arches.

In the case of clustered arches, which may be used to support a vault or dome, each semi-arch may be calculated as above. The horizontal thrust at the abutments will be equal to that at the crown of the arch. If it be required to determine the thrust all round any part of a dome, the following means may be used:

Fig. 12.



Let it be required to determine the total thrust upon the section *a b* of the dome shown in Fig. 12. Let i = the angle made by a line tangent to the dome in a vertical plane at *a* with the horizon, and W = the total load above the section *a b*, then will the total thrust be

$$= \frac{\mathrm{W}}{\sin i}$$

and the thrust per unit of the periphery will be, if d' = diameter at the joint

$$= \frac{\mathrm{W}}{\pi\,d'\sin i}$$

$$= \frac{\mathrm{W}}{3{\cdot}1416\,d'\sin i}$$

The horizontal radiating thrust will be

$$= \text{W cotan } i$$

and the horizontal thrust per unit of periphery

$$= \frac{\text{W cotan } i}{3{\cdot}1416\, d'}$$

If the tendency to spread at the joint *a b* be withstood by a hoop, the tension of such hoop will evidently be

$$= \frac{\text{W cotan } i}{3{\cdot}1416\, d'} \cdot \frac{d'}{2}$$

$$= \frac{\text{W cotan } i}{6{\cdot}2832}$$

PART III.

PRACTICAL.

In the present part of our treatise, it is proposed to recapitulate in a simplified form the results obtained in the second part, in order that they may be readily referred to when required for use, and the trouble of searching amongst the steps leading to them avoided; and, in addition to this, it is intended to intersperse such remarks of a practical character as may be likely to prove useful to those actually engaged in the construction of iron roofs.

The lattice girder under a uniform load first requires attention; (the same notations will be employed as were used in the theoretical part). The strain on any strut and the tie following it will be

$$= \frac{w \, . \, x \, . \, \lambda}{d \, n}$$

where x = the distance of the foot of the strut from the centre of the girder.

Suppose, for example, that the strain is to be determined on a strut and tie distant twelve feet from the centre of the girder, the weight of load being one ton and a half per lineal foot, with two series of triangles, the length of one lattice bar seven feet, and the depth of the lattice girder being five feet, the required strain will be

$$\frac{w \, . \, x \, . \, \lambda}{d \, n} = \frac{1 \cdot 5 \times 12 \times 7}{5 \times 2} = 12 \cdot 6 \text{ tons}$$

and taking 4 and 5 tons as the respective resistance of wrought iron to compression and tension, the nett sectional areas for that material will be

$$\text{For the strut } \frac{12 \cdot 6}{4} = 3 \cdot 15 \text{ sq. ins.}$$

$$\text{For the tie } \frac{12 \cdot 6}{5} = 2 \cdot 52 \text{ sq. ins.}$$

Fig. 3 exhibits a girder similar in construction to that here supposed.

It usually occurs in roof construction where lattice girders are employed, that either the Warren girder of one series shown in Fig. 1, or the girder with the diagonals at 45° to the horizon and two series, is chosen; wherefore it is desirable here to append formulæ for these special cases.

For the Warren girder, where the angle of the bars to the horizon is 60°, the formula for strain on any strut and tie becomes

$$= 1{\cdot}154 \,.\, w \,.\, x$$

so that the above example would, if it had related to a Warren girder, have been worked as under:

$$1{\cdot}154 \; w\,x = 1{\cdot}154 \times 1{\cdot}5 \times 12 = 20{\cdot}772 \text{ tons}$$

The strain is greater because there is but one series of triangles, and less in proportion to this, because the angle α is greater than in the former case.

The sectional areas of strut and tie will be

$$\text{For the strut } \frac{20{\cdot}772}{4} = 5193 \text{ sq. ins.}$$

$$\text{For the tie } \frac{20{\cdot}772}{5} = 4{\cdot}154 \text{ sq. ins.}$$

In the case of a two series girder, with bars at 45° to the horizon, the formula becomes

$$= 1{\cdot}414 \; w\,x$$

Working out the above example for such a girder, we find a strain on tie and strut

$$1{\cdot}414 \; w\,x = 1{\cdot}414 \times 1{\cdot}5 \times 12 = 25{\cdot}45 \text{ tons}$$

and the sectional areas

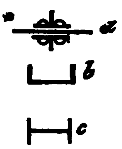
$$\text{Of the strut} = \frac{25{\cdot}45}{4} = 6{\cdot}36 \text{ sq. ins.}$$

$$\text{Of the tie } \frac{25{\cdot}45}{5} = 5{\cdot}09 \text{ sq. ins.}$$

The calculation of the nett areas may evidently be very readily performed by the foregoing simple formulæ, and it now remains for us to see how the material employed should be disposed in order to obtain satisfactory results.

Wrought iron appears to be the most suitable material for the purposes of which we treat; but whether wrought or cast iron be used, the form adopted will be substantially the same, the difference being that a greater number of joints will be required for wrought than for cast iron.

Fig. 13.



Tie bars, being subject to tensile strains only, may be formed of simple rectangular section, but struts must be made so as to offer as much resistance to flexure as possible. When cast iron is the material used, it is evident that the + section may conveniently be adopted for struts; but when wrought iron is employed one of the sections shown in Fig. 13 may be selected. *a* shows a section consisting of two T irons riveted back to back, with one or more plates interposed; *b* is a channel iron section, useful for girders of two series of triangles, as in such cases the struts crossing each other may be riveted together back to back at the point of intersection; *c* is a simple H section, which may frequently be employed with advantage, as offering greater rigidity than *b*, with less complicity than *a*.

With regard to the position of struts and ties in a lattice or triangular girder, it may perhaps be desirable here to insert the general law, which is "that under an uniformly distributed load all bars inclining from the centre of the girder downwards towards either point of support are struts, those bars which incline in the contrary direction being ties." The means of attaching the diagonal bars to the horizontal members will be subsequently mentioned.

Having disposed of the diagonal bars, we must proceed to consider the horizontal members.

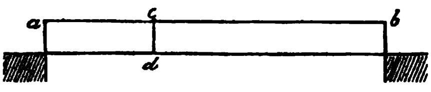
The means of determining the strains trigonometrically have already been described, but in practice it will be more convenient to use the following method. Let it be required to determine the strain between two consecutive joints of the bottom member with the diagonals; let x = the distance from one point of support to that one of the two chosen joints which is nearest the centre of the girder; let l = the distance between the extreme joints of the diagonals of the bottom member, w = the load per lineal foot upon the girder, and d = the depth of the girder; then will the required strain be

$$= \frac{w}{2}\frac{x}{d}\{l - x\}$$

The formula being obtained by regarding the distance x, and the depth of the girder, as forming a bent lever. *

It may be interesting to try how far the two methods agree when applied to a given example. Let w = 1 ton, the girder being of the form shown Fig. 1, page 3. Let the distance between two consecutive joints = 3 feet, and the space of the girder = 21 feet, there being 7 triangles, with diagonal bars placed at an angle of 60° to the horizon, suppose the strain is required on the bars between the second and third joints, it will be

$$
\begin{aligned}
&= 12{\cdot}5 \;.\; w\, s \;.\; \text{cotan}\ d \\
&= 37{\cdot}5 \times 0{\cdot}58 \\
&= 21{\cdot}75 \text{ tons.}
\end{aligned}
$$



* This formula may be thus explained. Let $a\,b$ (see margin) be a girder consisting of two flanges connected by a plate web, and let it be required to determine the strain at any point d, distant x from the point of support a. Let w = the load per lineal foot upon the girder, l = the span of the girder, D = the depth of the girder. We may consider $a\,c$, $c\,d$, as a bent lever acting about the fulcrum c; there will be two forces acting in opposite directions, the load between a and c pressing downwards will tend to force the point d towards b, the amount of this load will be

$$= w\,x$$

and its effective leverage

$$= \frac{x}{2\,\mathrm{D}}$$

hence the direct strain at the point d, due to this load, will be

$$= \frac{w\,x^2}{2\,\mathrm{D}}$$

There will also be another force acting upon d in the opposite direction, viz., the reaction of the pier a, which will be equal to the load upon it, which is

$$= \frac{w\,l}{2}$$

because each point of support bears half the total load. The effective leverage of this force will be

$$= \frac{x}{\mathrm{D}}$$

and the effect at d

$$= \frac{w\,l\,x}{2\,\mathrm{D}}$$

Let us now see what results we obtain by employing the more simple formula. Calculated by this method the strain will be

$$= \frac{1 \times 9}{2 \times 2 \cdot 66} \{21 - 9\}$$

$$= 12 \times \frac{9}{5 \, 33}$$

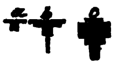
$$= 20 \cdot 26 \text{ tons}$$

A result somewhat less than that obtained by the preceding method, but nevertheless sufficiently accurate for practical purposes.

The method of calculating the top flange will be precisely the same as that given for the bottom flange, if $l =$ the distance between the extreme joints of diagonals with the top flange, from which it appears that in any triangular girder of the series there is a greater strain upon one flange than upon the other.

It yet remains to discuss the form of the horizontal members of lattice or triangular girders. The top member being subject to compressive strain, must evidently be made of such form as may appear most adequate to secure rigidity. Some sections suitable for light lattice girders are shown, Fig. 14. *a* and *b* are for wrought iron, the former consisting of two angle irons, between which the extremities of the diagonals may be riveted up; *b* is a section, consisting of a top plate, two angle irons, and a vertical plate between the latter, outside which the diagonals are riveted up, being placed upon opposite sides of the same; this will necessitate the insertion of packing pieces between the diagonals at their intersections, if they be there riveted together. The section beam may be modified by dispensing with either the top plate or the vertical plate, if necessary; but in the latter case it will be preferable to rivet up the extremities of the diagonals between the angle irons; *c* shows a section suitable for a cast iron compression member, which may safely be used for roof girders, when deemed otherwise expedient, as such girders are not liable to shocks or sudden variations of strain.

Fig. 14.



As the two foregoing forces act in opposite directions, their difference will be the effective strain at the point *d*, which will be

$$= \frac{w\,l\,x}{2\,\mathrm{D}} - \frac{w\,x^2}{2\,\mathrm{D}}$$

$$= \frac{w\,x}{2\,\mathrm{D}} \{l - x\}$$

which is the expression quoted in the text.

The bottom or tension member may conveniently be made of flat bars, similar to those used in the construction of suspension chains, or the sections *a* or *b* may be adopted, but cast iron should never be used for this portion of the structure.

In forming the junction of the diagonal bars with the flanges, one large pin or several small rivets or bolts may be employed, and the latter method is that which we think preferable, as the small holes will tend to weaken the metal less than one large one. It is expedient for appearance sake to cover the main joints of lattice girders used in roof structures with ornamental castings, whereby very pleasing effects may be obtained. In the formation of simple trusses, the main compression members may be of cast or wrought iron, with angle iron, T iron, or H iron section, and the ties of round or flat bars, the joints being made by means of pins passing through eyes in the extremities of the tie rods at the centre, and the end joints being made by passing the extremities of the tie rods through shoes attached to the extremities of the main compression member, and tightening them up by means of nuts fitted to the screwed ends of the tie bars.

The strains upon a simple upright or inverted truss of the form shown, Fig. 4, may be calculated readily by means of the following formulæ. Let the truss be horizontal and supported at each extremity, and let it be subject to a load, W, placed upon the centre; let l = the length of one tie rod, l' = the length of the centre strut, and L = the length of the main compression member; then will the strain on the central strut be

$$= W$$

that on one tie rod

$$= \frac{W}{2} \cdot \frac{l}{l'}$$

and that on the compression member

$$= \frac{W}{2} \cdot \frac{l}{l'} \cdot \frac{\frac{1}{2} L}{l}$$

$$= \frac{W L}{4 l'}$$

for the truss placed as shown in the Fig. When it is inverted the strains on each element are the same in intensity, but the horizontal member becomes a tie, the inclined members become struts, and the central vertical member is a tie if the load be applied at the bottom, and a strut if it be applied at the top. If the load be uniformly distributed, the above strains will be required to be halved. This form of truss, when used in the construction of roofs, is generally inverted.

In the truss shown in Fig. 5, the strains may be found from the following expressions for a distributed load. Let w = the load on one of the vertical members, l = the length of ditto, l' = the length of one of the inclined bars, then the strain on one vertical member will be

$$= w$$

that on one of the inclined bars

$$= w \cdot \frac{l'}{l}$$

on either of the horizontal members it will be

$$= w \cdot \frac{l'}{l} \cdot \frac{l''}{l'} = w \frac{l''}{l}$$

where l'' = distance of vertical member from nearest end of truss (= *d b*, Fig. 5).

This truss, like the last, is also generally used in an inverted position. In order to find the intensity of the strains upon the compound truss, Fig. 6, let W = the total distributed load, then will the load acting over any of the struts be

$$= \frac{W}{4}$$

the strain upon either of the bars inclining from the centre of one main inclined bar to the centre of the horizontal member will be

$$= \frac{W}{4} \cdot \frac{1}{4} \cdot \frac{l'}{l}$$

$$= \frac{W l'}{16 l}$$

where l' = half the length of one main inclined member (*g b*) and l = half the length of the centre vertical member (*d g*).

This occurs because the weight on one point (*e*) is divided between the piers, so as to act upon each with a force inversely as the distance.

The load upon the centre strut will be

$$= W\left\{\frac{1}{4} + \frac{1}{4} \cdot \frac{1}{4} + \frac{1}{4} \cdot \frac{1}{4}\right\}$$

$$= \frac{3\,W}{8}$$

and the strain on the lower part of either main inclined bar (*g f* or *g k*) will be

$$= \frac{3\,W}{2 \times 8} \cdot \frac{l'}{l} = \frac{3\,W\,l'}{16\,.\,l}$$

the strain upon the upper parts (*a f, h b*), of the main inclined bars will be, in addition to the foregoing, which is transmitted from the lower part, a stress

$$= \frac{W}{4} \cdot \frac{3}{4} \cdot \frac{l'}{l}$$

$$= \frac{3\,W\,.\,l'}{16\,.\,l}$$

hence the total strain upon these parts will be

$$= \frac{6\,W\,l'}{16\,.\,l} = \frac{3\,W\,.\,l'}{8\,.\,l}$$

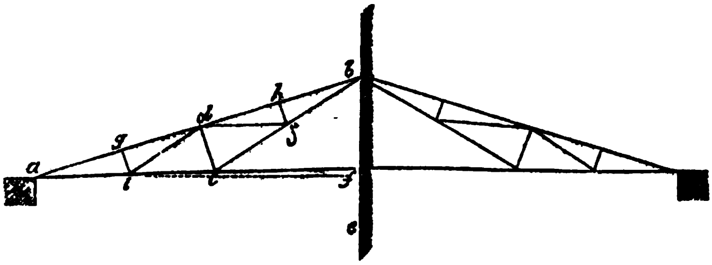
The thrust on the main horizontal member will be, if L = ½ span,

$$= \frac{3\,W\,.\,l'}{8\,.\,l} \cdot \frac{L}{l'}$$

$$= \frac{3\,W\,.\,L}{8\,.\,l}$$

By following out a course similar to the above, the strains upon much more complicated trusses may with equal facility be resolved. The same formulæ may, with a very slight modification, be also applied to inclined trusses, of which we will now proceed to treat an example.

Fig. 15.



Let *a, b, c*, Fig. 15, represent an inclined truss resting against a wall, *b e*. It is evident that we may replace this wall by another truss, as shown, without vitiating the principle of the structure; and the two trusses may be united with a tie, in order to withstand the tendency of the two trusses to spread or separate from each other. We may then proceed to calculate each half of the structure as a separate inclined truss.

Let W be the load on $a\ b$ (half the roof), then we may consider that on each truss there is a load

$$= \frac{W}{4}$$

on the three intermediate struts, and a load

$$= \frac{W}{8}$$

at the foot and apex.

Let $d =$ the total rise of the roof; that is to say, the height of the truss, $f\ b$ in the figure. Let $L = a$ ~~k~~ b; then will the strains upon different parts of the main compression member be as follows:—

$$\text{On } b\ k \text{ a strain} \quad = C + \frac{W}{8} \cdot \frac{L}{d}$$

where $c =$ the strain upon $a\ b$ by the main ties.

$$\text{On } k\ d \text{ a strain} = C + \frac{W L}{8\ d} + \frac{W L}{4\ d}$$

$$= C + \frac{3\ W\ L}{8\ .\ d}$$

$$\text{On } d\ g \text{ a strain} = C + \frac{3\ W\ L}{8\ .\ d} + \frac{W L}{4\ d}$$

$$= C + \frac{5\ W\ L}{8\ .\ d}$$

$$\text{On } g\ a \text{ a strain} = C + \frac{5 W\ .\ L}{8\ d} + \frac{W L}{4\ d}$$

$$= C + \frac{7\ W\ .\ L}{8\ d}$$

$$\text{And at the point } a \text{ a strain} \quad = C + \frac{7\ W\ L}{8\ .\ d} + \frac{W L}{8\ d}$$

$$= C + \frac{W\ .\ L}{d}$$

The value of c will subsequently be determined

The next step will consist in the determination of the thrusts upon the struts, $k\ j$, $d\ c$, and $g\ i$.

Following out the same course as in the theoretical part, but with simple notations, we find that the strain upon either *k j* or *g i*, will be

$$= \frac{W}{4} \cdot \frac{L}{S}$$

where S = half the span of roof = *a f*.
The strain upon the strut, *d c*, will be

$$= \frac{W L}{4 S} + \frac{W L}{4 S}$$
$$= \frac{W L}{2 S}$$

the thrusts being produced partly by the load upon the point *d*, and partly by the loads on *g* and *k*. It yet remains for us to determine the strains upon the various ties in the truss.

Resolving the strains due to each part of the load, we find that the tie bars are subject to the following:—

The strain on *j d*, or *d i*, is

$$= \frac{W L}{8 S} \cdot \frac{l'}{l}$$

where $l' = j\,d$ and $l = k\,j$;
the strain upon *c j* will be

$$= \frac{W L}{4 . S} \cdot \frac{l'}{l}$$

that upon *j b*

$$= \frac{W . L . l'}{4 . S . l} + \frac{W . L . l'}{8 . S . l}$$
$$= \frac{3 W . L . l'}{8 . S . l}$$

hence the value of *c* will be

$$C = \frac{3 . W . L . l'}{8 . S . l} \cdot \frac{L}{4 . l'}$$
$$= \frac{3 W . L^2}{8 . S . l}$$

and *c* may be replaced by this value in the previous equations, in order to find the actual stress upon the various parts of the main compression member, *a b*.

The tension upon $c\,f$ will be $= \dfrac{\mathrm{W\,S}}{2\,d}$

and the tie $a\,c$ will have to sustain, in addition to this horizontal strain, strains equivalent to those upon the bar $c\,b$.

The horizontal strain will of course be slightly modified in $a\,c$, because the latter is at an angle to the horizon; but the above value will nevertheless be sufficiently accurate for all practical purposes, and so assuming it the strains will be

Upon $c\,i$ a strain $= \dfrac{\mathrm{W}\,.\,\mathrm{L}\,.\,l'}{4\,.\,\mathrm{S}\,.\,l} + \dfrac{\mathrm{W\,S}}{2\,d}$

And upon $i\,a$ a strain $= \dfrac{3\,.\,\mathrm{L}\,.\,l'}{8\,.\,\mathrm{S}\,.\,l} + \dfrac{\mathrm{W\,S}}{2\,d}$

This completes the resolution of the strains upon every bar in one truss, or half the roof; and, as the strains are symmetrical, it is unnecessary to calculate those upon the other truss. The foregoing example has been chosen not so much as a special case, as on account of its exhibiting well the *method* to be followed in solving the strains upon any complex truss or series of trusses.

In all cases tied roofs (*i.e.*, roofs which are tied at their lower extremities) exert only a direct vertical pressure upon the supporting walls, equal to the total load upon the roof.

The trusses being placed at given distances (say x feet) apart to support a roof whose weight is w lbs. per square foot, and whose semi-span is S, the load upon each half truss or half principal will necessarily be

$$\mathrm{W} = w\,.\,\mathrm{S}\,.\,x$$

which is the value to be substituted for W in the foregoing expressions when the other quantities are known.

The strains upon each element being known, the areas may be easily determined by the following simple expressions. Let s = the strain in tons upon any tie or strut, s' = the strain in lbs. upon any tie or strut, and a the nett sectional area of any tie or strut in square inches, then for tensile strain the sectional area will be

$$a = \frac{\mathrm{S}}{5}$$

or

$$a = \frac{S'}{11200}$$

for compressive strain, the least sectional area will be

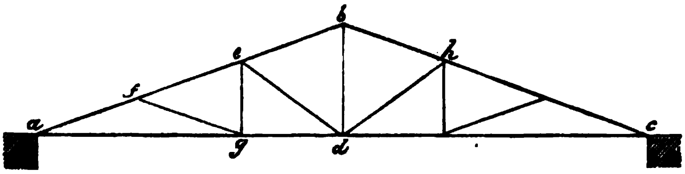
$$a = \frac{S}{4}$$

or

$$a = \frac{S'}{8960}$$

Let us now pass on to consider an example of another class of roof.

Fig. 16.



Let a b c represent a principal of roof; it consists of a primary truss, a b c, and of four secondary trusses, two on each side (a f g, a e d.) Let it be required to determine the strains upon each element of one half of the principal, W being = the weight to be supported by a b. Let a b = L, a d = S, b d = d. The bar a b being divided into three equal parts by the points f and e, so that $af = fe = eb$, then the load may be regarded as supported at the four points, a, f, e, b, the distribution of weight upon these points being

$$= \frac{W}{3}$$

upon each of the points f and e, and

$$= \frac{W}{6}$$

upon each of the points a and b.

The strain produced upon the rafter at a, due to the load upon that point, will be

$$= \frac{W}{6} \cdot \frac{L}{d}$$

which will be sustained in addition to others transmitted from higher parts of the rafter.

The load at *f* will be transmitted half direct to *a* and half to *g*, the strains thereby produced on *f a*, *f g*, being each

$$=\frac{W}{6}\cdot\frac{L}{d}$$

At *e* there will be the load there accumulated, and the load transmitted through the rod *g e*, together

$$=\frac{W}{3}+\frac{W}{6}=\frac{W}{2}$$

Of this load one-third will be transmitted to *a*, and two-thirds to *d*, the thrusts produced thereby being

On *e f* a strain $\frac{W}{2}\cdot\frac{1}{3}\cdot\frac{L}{d}$

$$=\frac{W\,L}{6\,d}$$

On *e d* a strain $=\frac{W}{2}\cdot\frac{2}{3}\cdot\frac{L}{d}$

$$=\frac{W\,L}{3\,d}$$

At *b* there will be a load

$$=\frac{W}{6}+\frac{W}{3}=\frac{W}{2}$$

This last load $\left(\frac{W}{3}\right)$ being transmitted through that half of the bar *b d* which may be regarded as belonging to the half truss now under consideration.

The strain produced at *b* upon *b a* will be

$$=\frac{W}{2}\cdot\frac{L}{d}$$

Summing the strains upon the rafter *a b*, we find

On *b e* a strain $=\frac{W\,L}{2\,d}$

On *e f* a strain $=\frac{W\,L}{2\,d}+\frac{W\,L}{6\,d}$

$$=\frac{2\,W\,L}{3\,d}$$

On $f\,a$ a strain $= \frac{2\,W\,L}{3\,d} + \frac{W\,L}{6\,d}$

$= \frac{5\,W\,L}{6\,d}$

At the point a a strain $= \frac{5\,W\,L}{6\,d} + \frac{W\,L}{6\,d}$

$= \frac{W\,L}{d}$

The strains upon the struts are already given, and those upon the vertical ties are equal to the loads supported by the same, which are

On $e\,g$ a load $= \frac{W}{6}$

And on half $d\,b$, a load $= \frac{W}{3}$

It yet remains for us to determine the pull upon each part of the main tie, the strains upon which will be as follows:—

Upon $a\,g$ a strain $= \frac{W\,L}{d} \cdot \frac{S}{L} = \frac{W\,S}{d}$

On $g\,d$ a strain $= \frac{W\,S}{d} - \frac{W\,S}{6\,d} = \frac{5\,W\,S}{6\,d}$

and between the junctions of the bars, $d\,e$, $d\,h$, with a strain

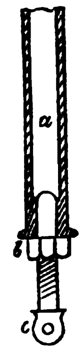
$$= \frac{5\,W\,S}{6\,d} - \frac{2\,W\,S}{6\,d} = \frac{W\,.\,S}{2\,d}$$

The strains upon each element of the roof being known, the areas of those elements may be readily determined by means of the expressions already given for sectional areas.

The foregoing examples being sufficient to illustrate the methods of calculating the strains upon roof structures, we shall next proceed to make a few remarks relative to the forms of the elements of the same.

Struts, or compression members forming the rafters may conveniently be made of angle or T iron; and the vertical or oblique struts may be made of T iron, or of iron of X section, and the ties may be formed of round or flat bars.

Fig. 17.



It is very necessary that the lengths of the various elements of a roof truss be accurate, in order to ensure satisfactory results; and it is therefore desirable to have some means of adjustment, which may be supplied to the struts by adopting some form of construction shown in Fig. 17. *a* shows the lower extremity of the central part of a strut; *c* is the eye, connecting the struts with the neighbouring elements; the eye, *c*, is prolonged; the upper part of the prolongation being accurately turned to fit the lower extremity of the central part of the strut, which is bored. The eye-piece has also a thread cut upon it, and is furnished with a nut, *b*, by which the length of the strut can be adjusted. The central part of the strut then bears upon the nut, but if, through an improper adjustment of the parts, a tensile strain comes upon this arrangement, it is exhibited by the part *a* leaving its bearing upon the nut *b*, which must then be screwed up until the adjustment is correct.

The various extremities of the elements may be united by means of links or bored plates and pins, and the means are too obvious to require further comment.

Having dispensed with lattice girders and trusses, arches now require our attention. In practice, it will only be necessary to determine the thrust at the crown and haunch of the arch, which may readily be done by means of the following simple expressions:—

Let W = equal the load on half the arch, L = half the span of the arch, V = the rise or versine of the arch, the load being uniformly distributed over it. The thrust at the crown of the arch will then be

$$= \frac{W \,.\, L}{2\,V}$$

and that at the haunch

$$= \sqrt{\frac{W^2 . L^2}{4\,V^2} + W^2}$$

$$= W\sqrt{\frac{L^2}{4\,V^2} + 1}$$

Let L = $n \,.\, v$, then the foregoing formulæ become, for the crown

$$= \frac{W\,n}{2}$$

and for the haunch

$$= W\sqrt{\frac{n^2}{4} + 1}$$

The arch may (if of wrought iron) conveniently be made of a plate with angle iron flanges top and bottom.

Domes only now remain to be treated. If there be a dome supported by a group of half arches, each may be calculated as such; each half arch supporting a portion of the roof equal to that which is contained between two successive semi-arches.

If, however, the dome be of uniform construction throughout, it may be calculated according to the formulæ exhibited in the theoretical part, or it may be supposed to consist of an indefinite number of semi-arches placed close together side by side, each semi-arch having a given breadth at the springing and diminishing to nothing at the crown.

We have now concluded our remarks upon the construction of iron roofs, but think it desirable here to observe that the calculations given in the preceding pages with regard to the determination of strain may with equal accuracy be applied to timber roofs; but it will, of course, be necessary in that case to use different co-efficients of strength, in order to determine the sectional areas of the elements, because of the difference of strength in iron and timber, the constant depending upon the quality of the material.

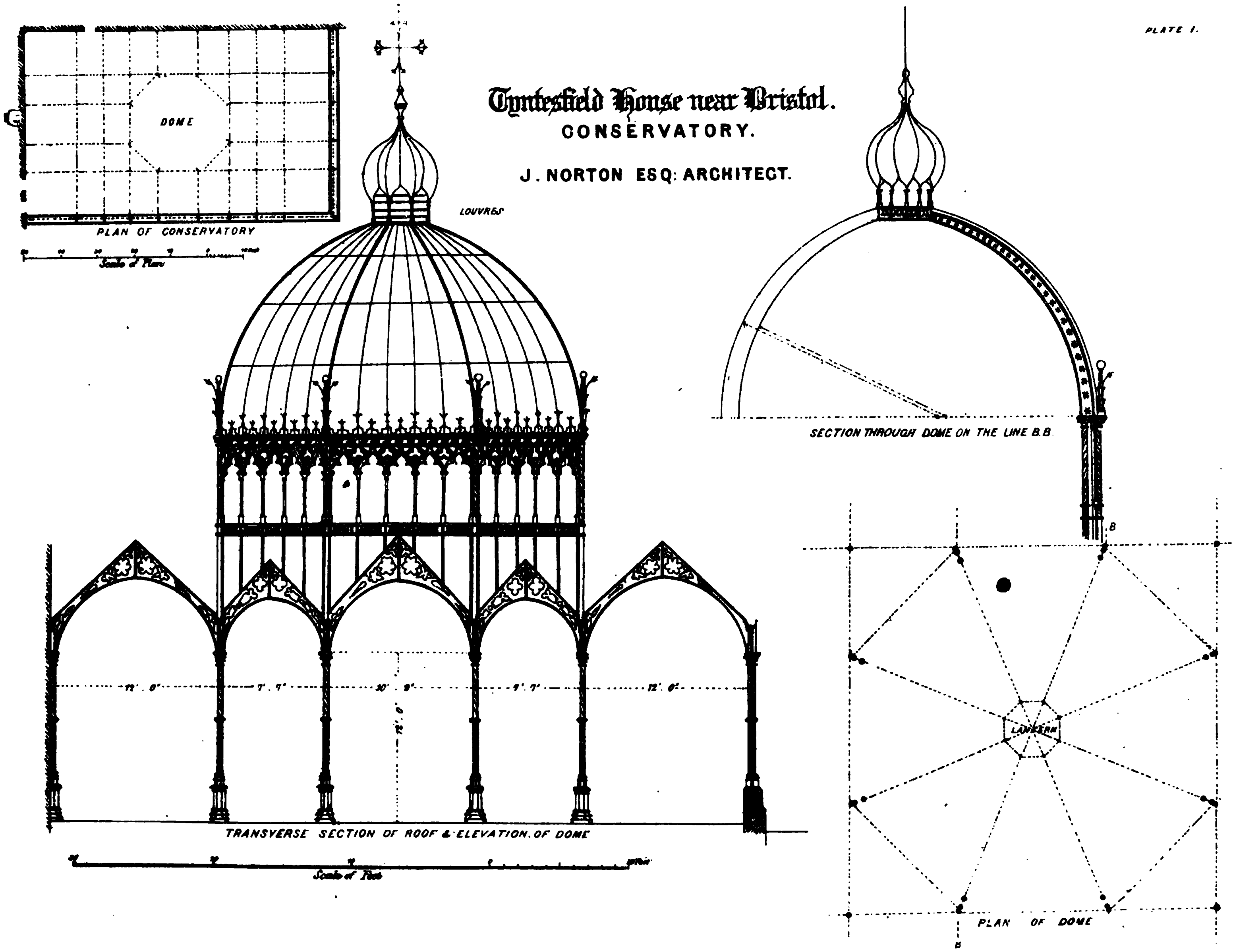
THE END.

PLATE I.

Tyntesfield House near Bristol.

CONSERVATORY.

J. NORTON ESQ: ARCHITECT.



PLAN OF CONSERVATORY

TRANSVERSE SECTION OF ROOF & ELEVATION. OF DOME

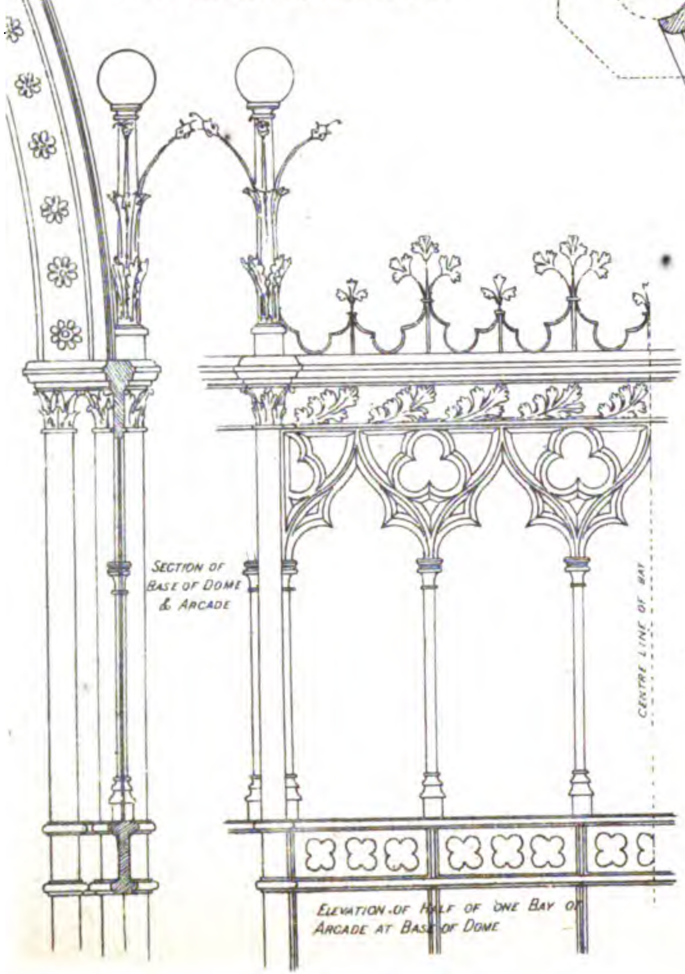
SECTION THROUGH DOME ON THE LINE B.B.

PLAN OF DOME

Tyntesfield House near Bristol.

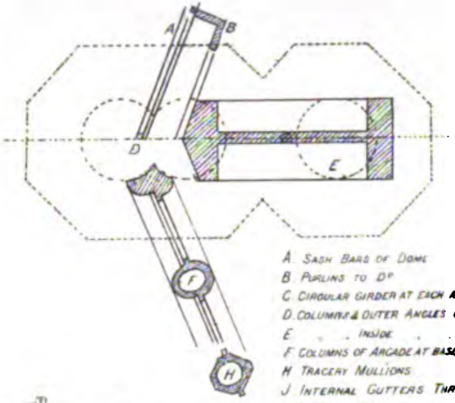
DETAILS OF CONSERVATORY.

J. NORTON ESQ: ARCHITECT

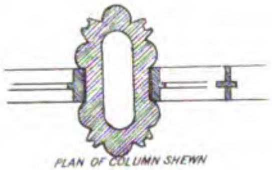


SECTION OF BASE OF DOME & ARCADE

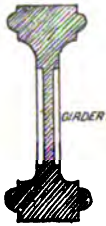
ELEVATION OF HALF OF ONE BAY OF ARCADE AT BASE OF DOME



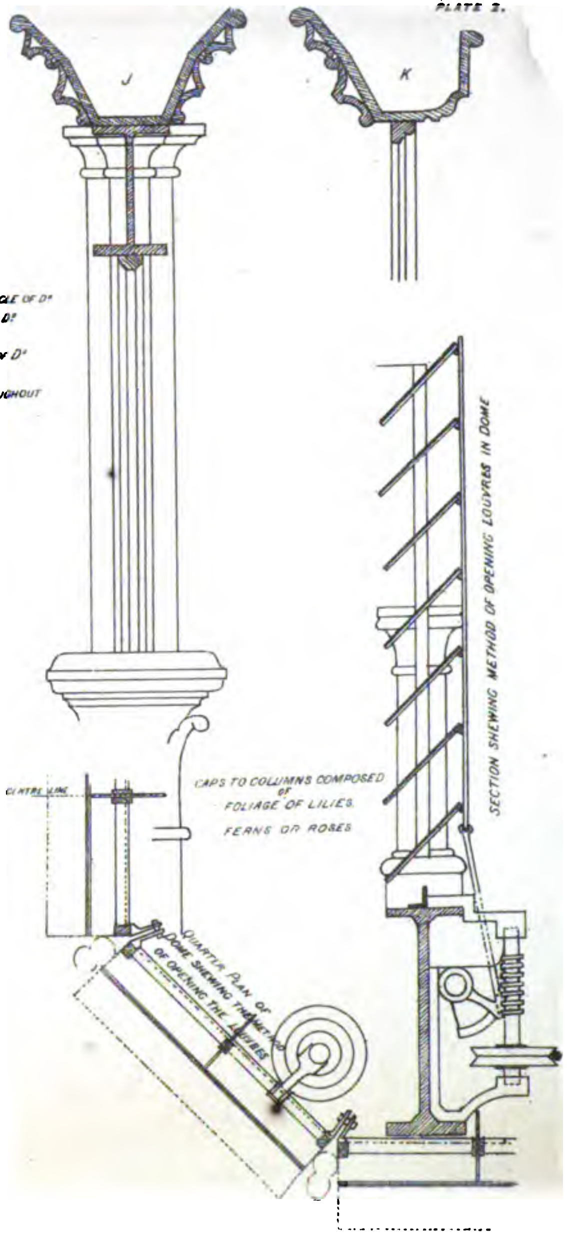
A. Sash Bars of Dome
B. Purlins to D°
C. Circular Girder at each angle of D°
D. Columns & Outer Angles of D°
E. . . Inside .
F. Columns of Arcade at base of D°
H. Tracery Mullions
J. Internal Gutters throughout
Conservatory
K. External Gutters to D°



PLAN OF COLUMN SHEWN ABOVE AT E



GIRDER AT L



CAPS TO COLUMNS COMPOSED OF FOLIAGE OF LILIES FERNS OR ROSES

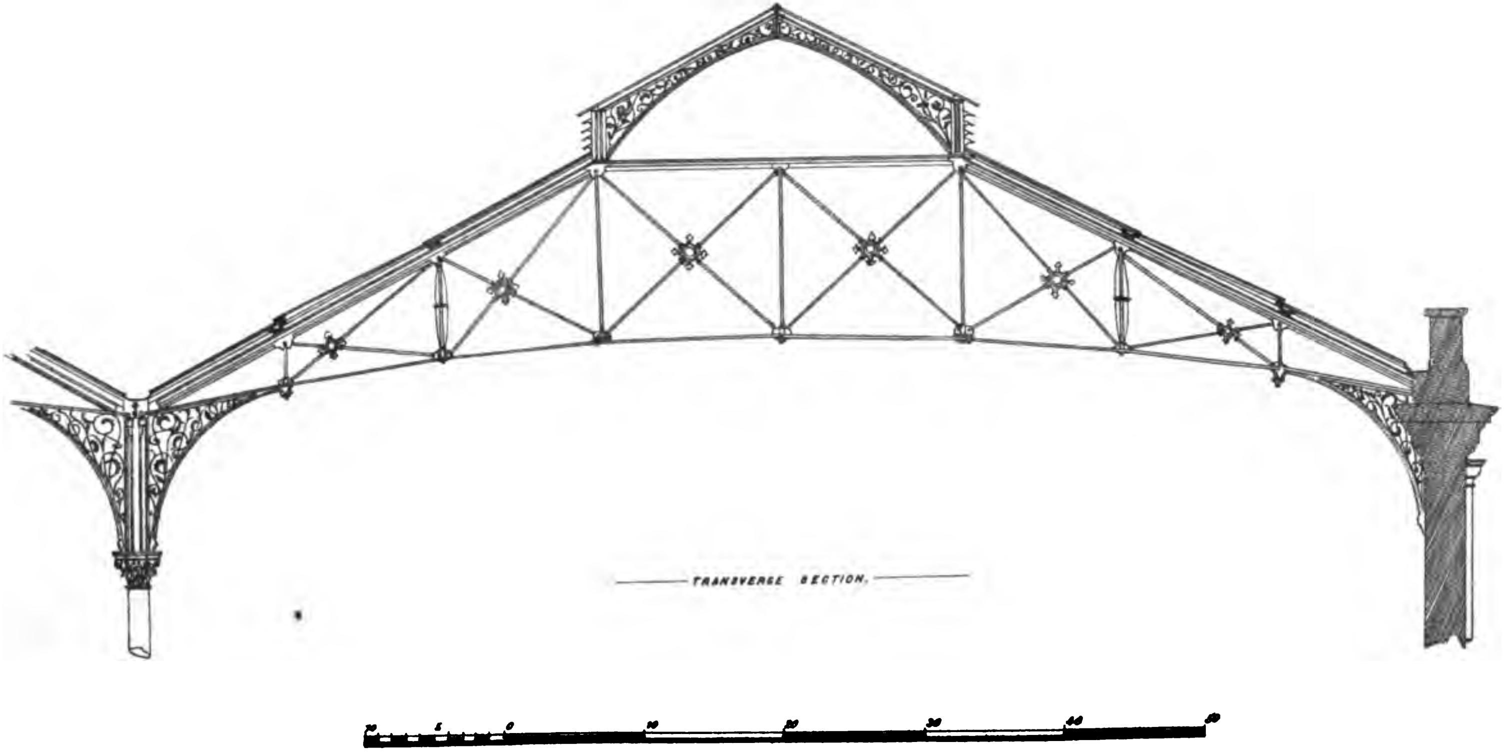
QUARTER PLAN OF DOME SHEWING THE METHOD OF OPENING THE LOUVRES

SECTION SHEWING METHOD OF OPENING LOUVRES IN DOME

London & North Western, & North London Railways,

BROAD STREET STATION,

IRON ROOF TO STATION.
Wm BAKER Esqre ENGINEER.

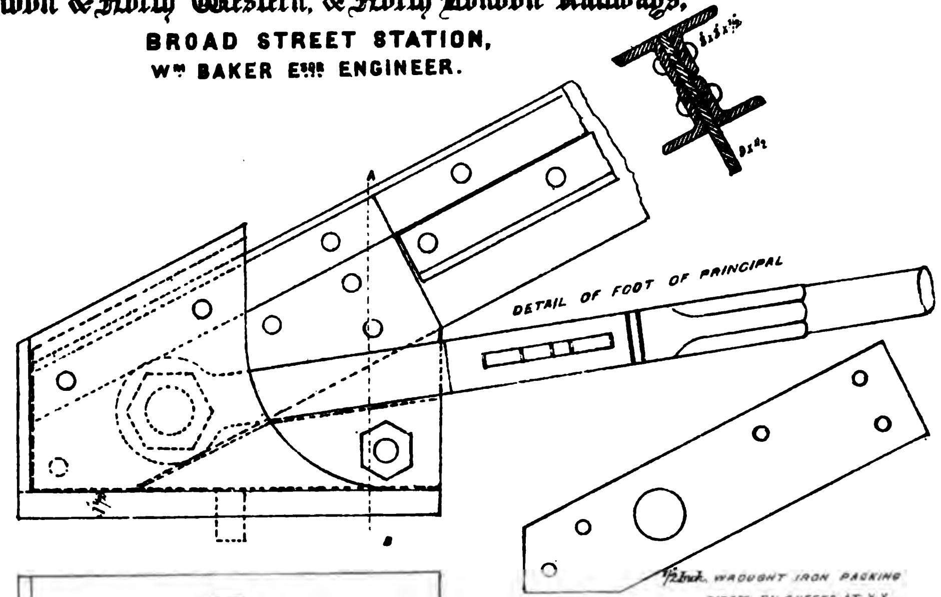


TRANSVERSE SECTION.

London & North Western, & North London Railways,

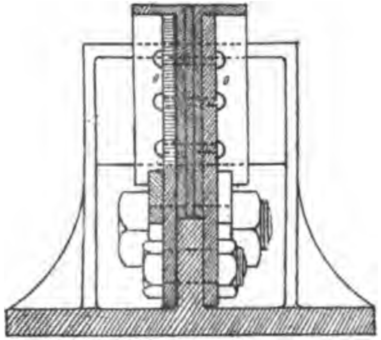
BROAD STREET STATION,

Wm BAKER Esqr ENGINEER.

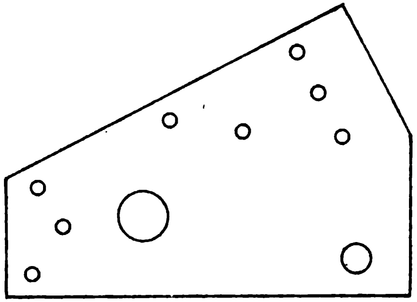


DETAIL OF FOOT OF PRINCIPAL

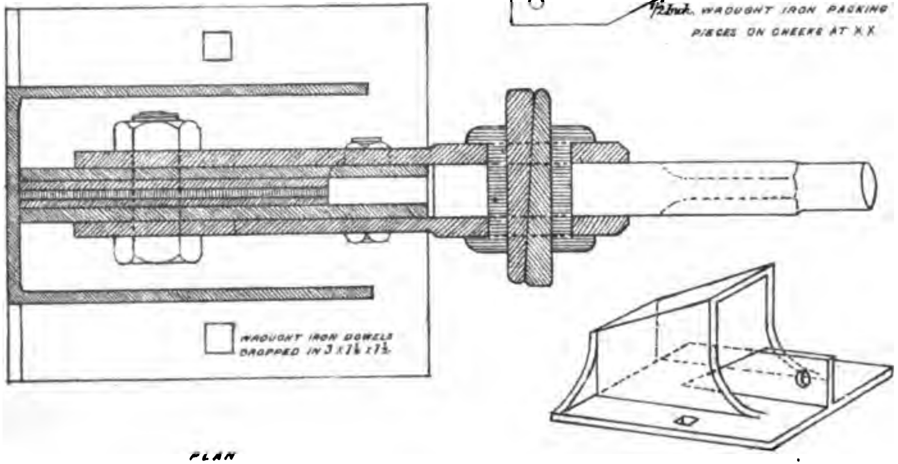
½ Inch WROUGHT IRON PACKING PIECES ON CHEEKS AT X.X.



SECTION AT—A—B.

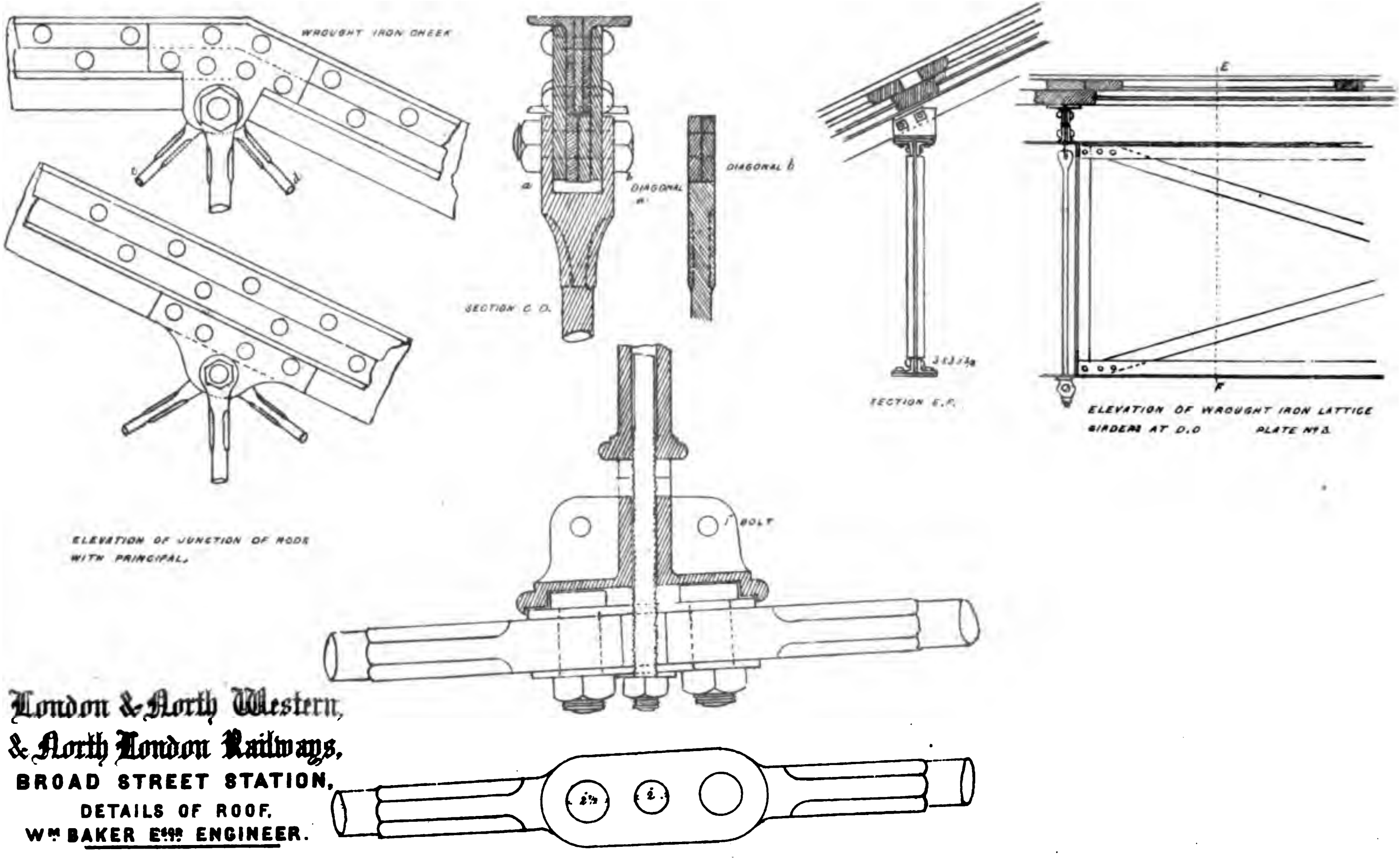


3/4 Inch WROUGHT IRON CHEEKS AT O.O.



PLAN

VIEW OF CAST IRON SHOE.



ELEVATION OF JUNCTION OF RODS WITH PRINCIPAL.

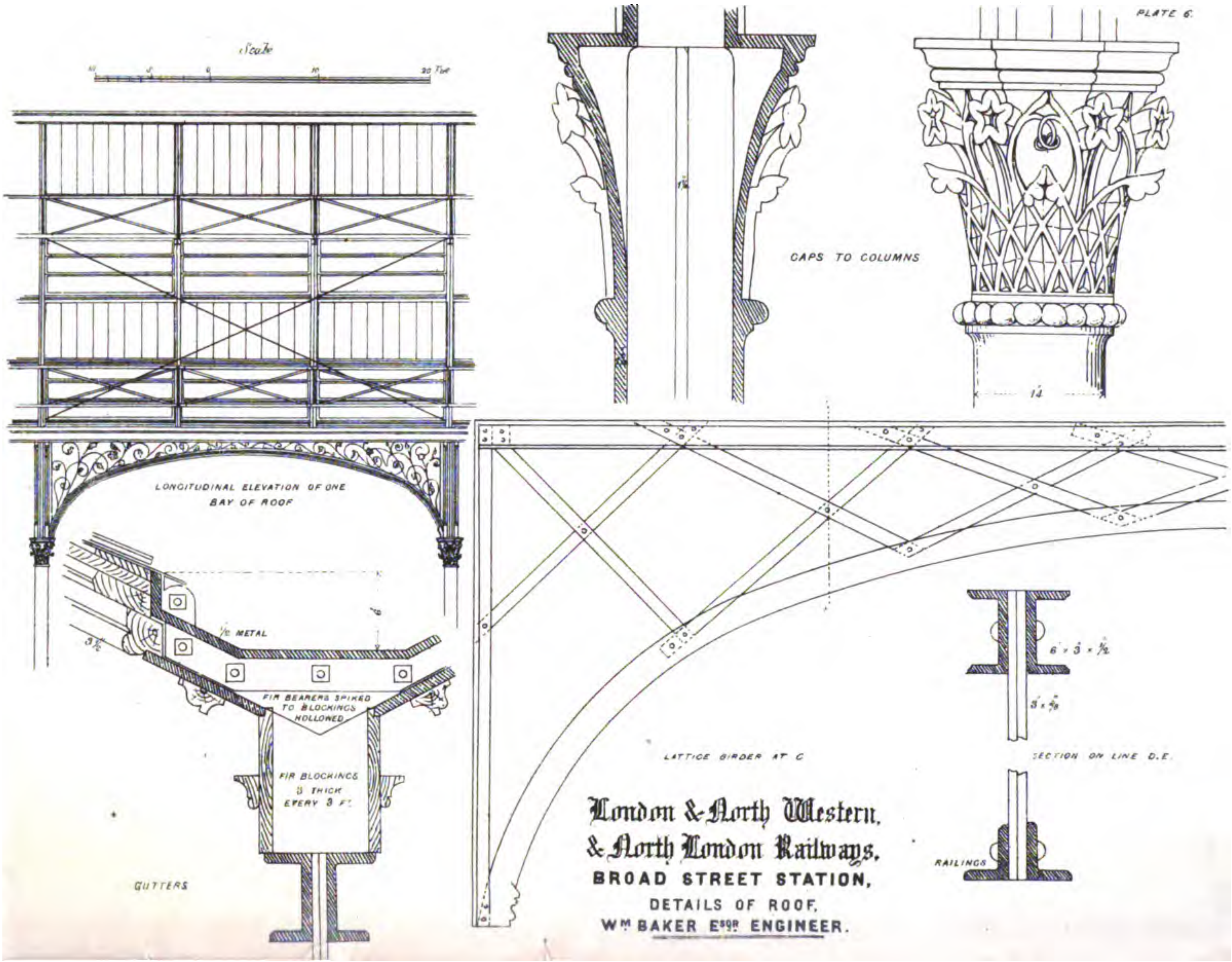
SECTION C.D.

SECTION E.F.

ELEVATION OF WROUGHT IRON LATTICE GIRDERS AT D.D   PLATE Nº 3.

DETAIL AT A PLATE 3.

London & North Western,
& North London Railways,
BROAD STREET STATION,
DETAILS OF ROOF.
Wᴹ BAKER Esqʳᵉ ENGINEER.



London & North Western,
& North London Railways,
BROAD STREET STATION,
DETAILS OF ROOF.
Wm BAKER Esqr ENGINEER.

PLATE 7.

Charing Cross Railway,

CITY TERMINUS ROOF,

JOHN HAWKSHAW ESQ.RE ENGINEER.

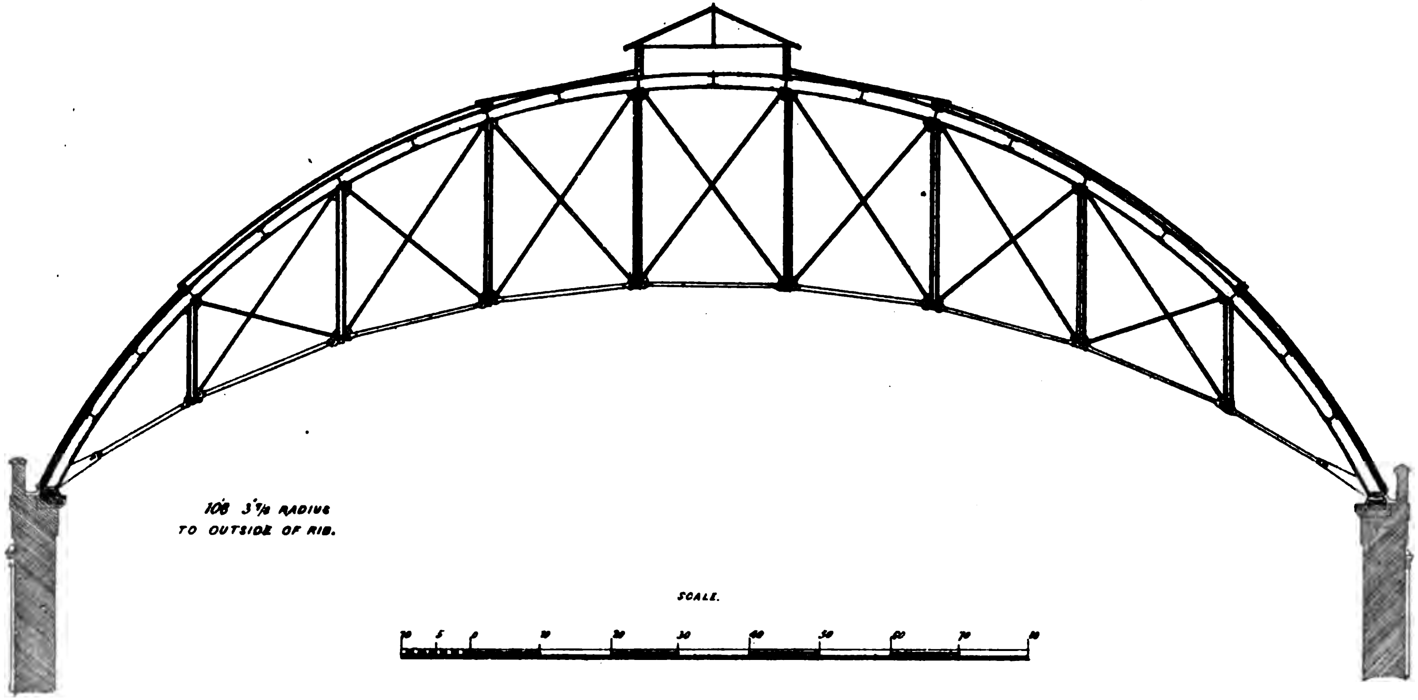
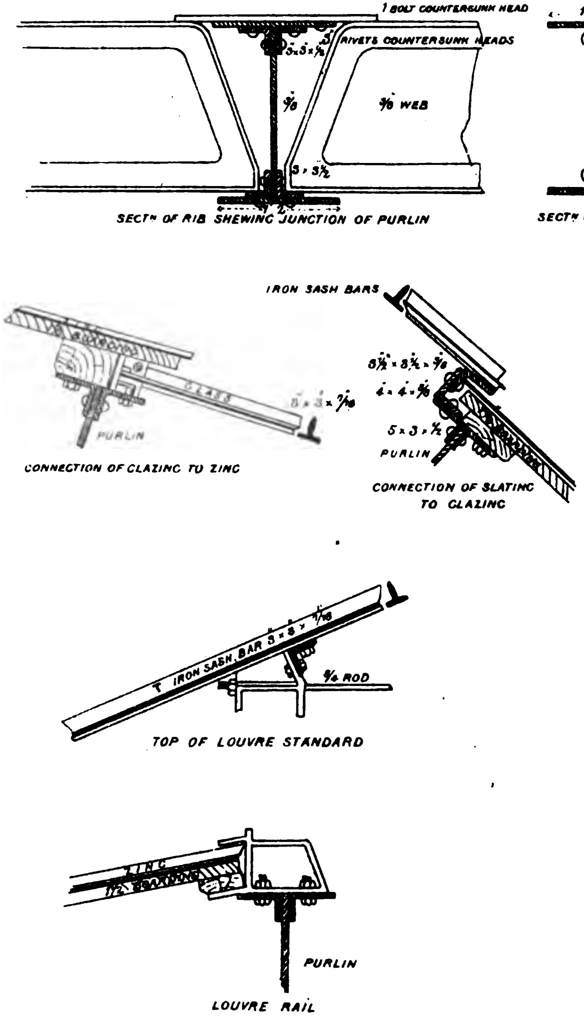
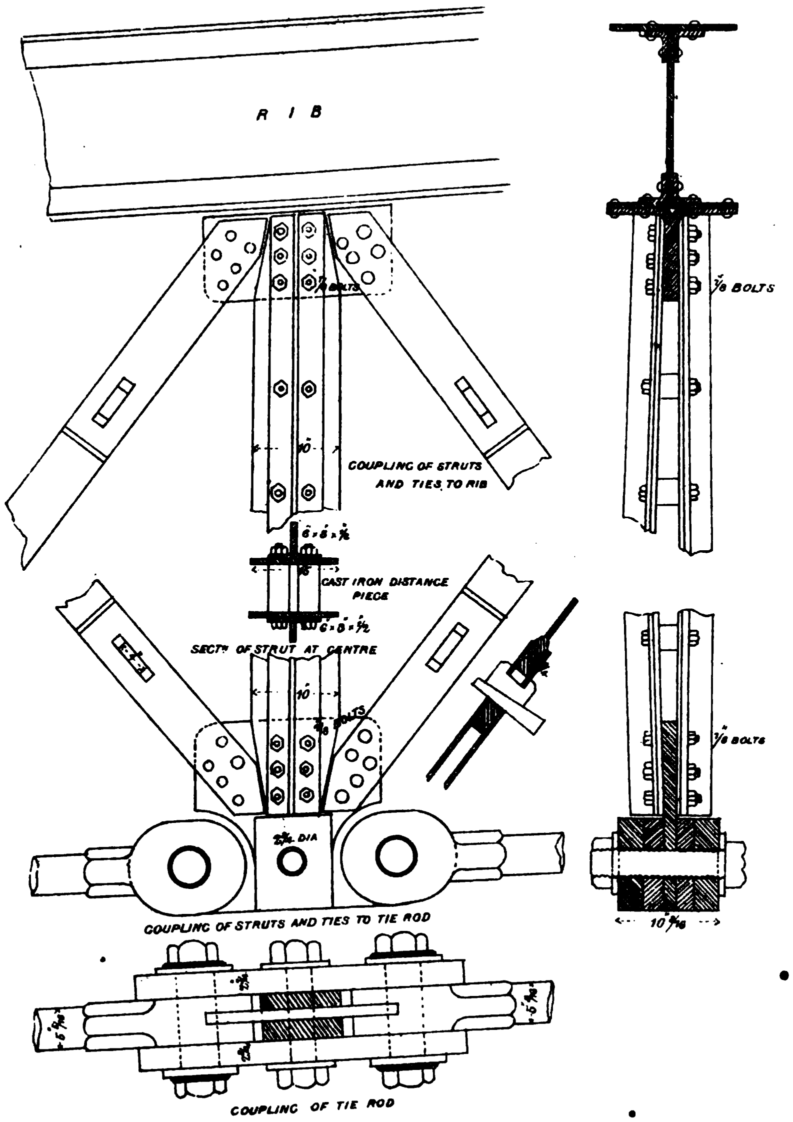


PLATE 8.





Charing Cross Railway.

DETAILS OF CITY TERMINUS ROOF.

J. HAWKSHAW ESQRE ENGINEER.

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